CIRCUITS AND SYSTEMS LABORATORY

EXERCISE 1

SINGLE-PHASE RLC CIRCUITS AT SINUSOIDAL EXCITATION

1. PROGRAM OF EXPERIMENTS

1.1. *RLC* CIRCUITS WITH NO MAGNETIC COUPLING

The experiments program of *RLC* circuits cover the following:

- examining the serial *RLC* circuit,
- examining the parallel G*LC* circuit,
- examining the mixed serial and parallel *RLC* circuit.

Special panels, as presented in figure 1.1. with R, L and C elements are provided to conduct the research. The panels contain real elements, i.e. resistors, inductors and capacitors. Elements built into panels may be combined into any wanted serial and/or parallel combination. The teacher running the class is to give instruction on which choice of configuration (a, b or c) is to be chosen so that modules of impedance of each element are close.

Selected choice of elements' configuration is to be written in table 1.1 together with approximate values of elements.

Measurements are conducted with use of the circuits assembled as shown in figure 1.2, where the same configuration of supply part consisting of voltage source U_z and meters is used each time in combination with serial *RLC*, parallel *GLC* and one of the mixed serial and parallel circuits (c or d). The results of measurements are to be written into appropriate tables: 1.2, 1.3, 1.4, 1.5.

The alternating voltage supply provided in the laboratory allows selection of one of four voltage values: 11, 22, 33 or 44 V. The voltage should be set up to assure that the value of the current flowing into the circuit is between 0,5 and 1,5 A. Parameters of the elements used in the exercise are to be calculated according to the measurements.

Table 1. 1

Indication of chosen elements configuration and their values

R	a	b	c	$R\cong$
L	a	b	c	$X_L\cong$
C	a	b	c	$X_C\cong$

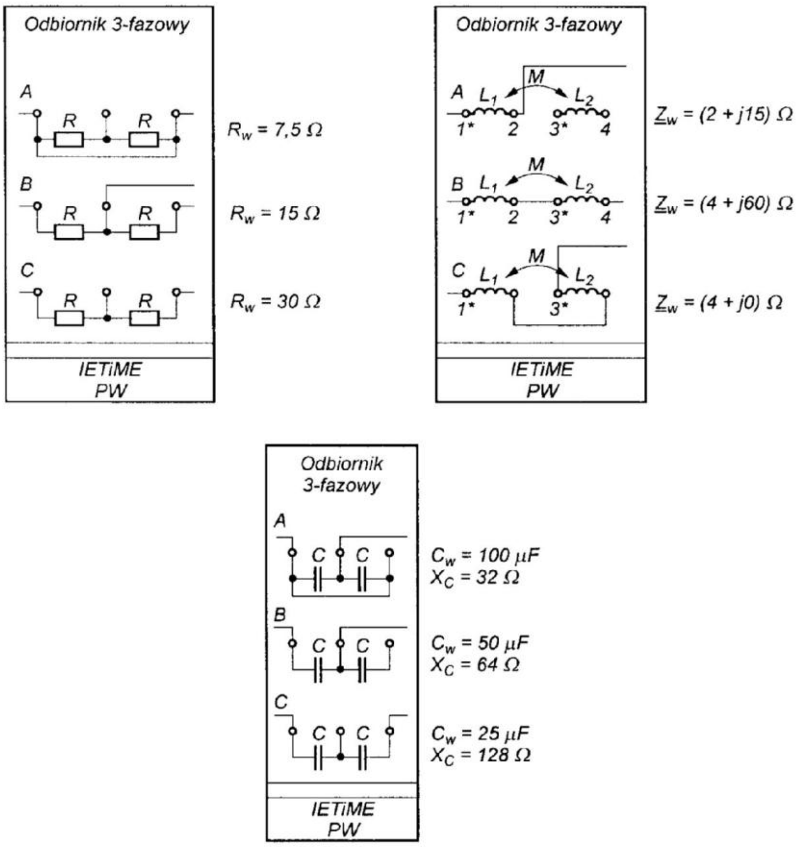

Fig. 1. 1. Elements configuration diagrams shown on the panels

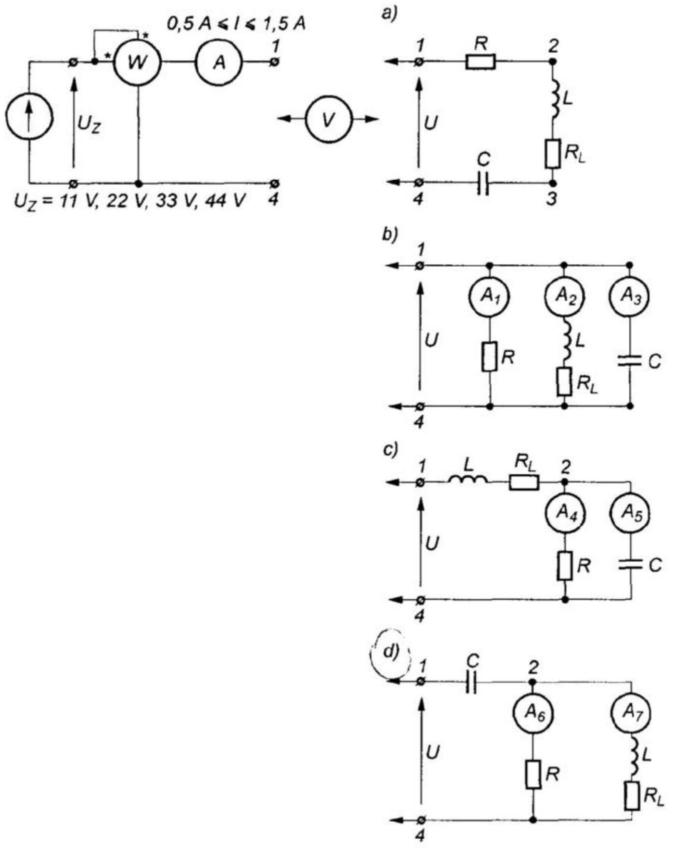

Fig. 1. 2. Circuits diagrams for examination

Table 1.2

Results table for diagram a

Measurements								Calculations													
U_Z	I	P_{14}	P_{12}	U_{12}	U_{23}	U_{34}	U_{14}	R	P_{23}	R_L	Z_L	X_L	$\underline{Z}_L$	X_C	$\underline{Z}_W$	$\cos\varphi_W$	φ_W	S	Q
V	A	W	W	V	V	V	V	Ω	W	Ω	Ω	Ω	Ω	Ω	Ω	–	°	VA	var

where:

$$9R = \frac{P_{12}}{I^2}, \quad P_{23} = P_{14} - P_{12}, \quad R_L = \frac{P_{23}}{I^2}, \quad Z_L = \frac{U_{23}}{I}, \quad X_L = \sqrt{Z_L^2 - R_L^2},$$

$$S = U_{14} I, \quad Q = \sqrt{S^2 - P_{14}^2}, \quad \underline{Z}_W = R + R_L + j(X_L - X_C) = R_W + jX_W, \quad \cos\varphi_W = \frac{R_W}{Z_W}$$

Table 1.3

Results table for diagram b

Measurements							Calculations											
U_Z	I	P	I_1	I_2	I_3	U_{14}	R	P_L	R_L	Z_L	X_L	U_{RL}	U_{XL}	X_C	Z_{14}	$\cos\varphi_W$	φ_W	S	Q
V	A	W	A	A	A	V	Ω	W	Ω	Ω	Ω	V	V	Ω	Ω	–	°	VA	var

where:

$$P_R = RI_1^2, \quad P_L = P - P_R, \quad R_L = \frac{P_L}{I_2^2}, \quad U_{RL} = R_L I_2, \quad U_{XL} = X_L I_2,$$

$$\underline{Y}_{14} = \frac{1}{R} + \frac{1}{R_L + jX_L} + \frac{1}{-jX_C}, \quad \underline{Z}_{14} = \frac{1}{\underline{Y}_{14}} = R_{14} + jX_{14}, \quad \cos\varphi_W = \frac{R_{14}}{Z_{14}}$$

Table 1.4

Results table for diagram c

Measurements									Calculations												
U_Z	I	P_{14}	P_{12}	I_4	I_5	U_{12}	U_{24}	U_{14}	R_L	Z_L	X_L	R	X_C	P_R	$\underline{Z}_{24}$	$\underline{Z}_{14}$	$\cos\varphi_W$	φ_W	S	Q
V	A	W	W	A	A	V	V	V	Ω	Ω	Ω	Ω	Ω	W	Ω	Ω	–	°	VA	var

where:

$$R_L = \frac{P_{12}}{I^2}, \quad Z_L = \frac{U_{12}}{I}, \quad X_L = \sqrt{Z_L^2 - R_L^2}, \quad R = \frac{U_{24}}{I_4}, \quad P_R = P_{14} - P_{12},$$

$$\underline{Y}_{24} = \frac{1}{R} + \frac{1}{-jX_C}, \quad \underline{Z}_{24} = \frac{1}{\underline{Y}_{24}}, \quad \underline{Z}_{14} = \underline{Z}_L + \underline{Z}_{24} = R_W + jX_W, \quad \cos\varphi_W = \frac{R_W}{Z_{14}}$$

Table 1.5

Results table for diagram d

Measurements									Calculations													
U_Z	I	P_{14}	P_{12}	I_6	I_7	U_{12}	U_{24}	U_{14}	R	P_R	P_L	R_L	Z_L	X_L	X_C	$\underline{Z}_{24}$	$\underline{Z}_{14}$	$\cos\varphi_W$	φ_W	S	Q
V	A	W	W	A	A	V	V	V	Ω	Ω	Ω	Ω	Ω	W	Ω	Ω		–	°	VA	var

where:

$$R = \frac{U_{24}}{I_6}, \quad P_R = RI_6^2, \quad P_L = P_{14} - P_R, \quad R_L = \frac{P_L}{I_7^2}, \quad Z_{12} = \frac{U_{12}}{I},$$

$$\underline{Y}_{24} = \frac{1}{R} + \frac{1}{R_L + jX_L}, \quad \underline{Z}_{24} = \frac{1}{\underline{Y}_{24}}, \quad \underline{Z}_{14} = \underline{Z}_{12} + \underline{Z}_{24} = R_W + jX_W, \quad \cos\varphi_W = \frac{R_W}{Z_{14}}$$

Values of resistance, actual inductor impedance and capacitor reactance calculated in experiments with serial, parallel and mixed serial and parallel circuits are to be written in table 1.6. The average value should be computed then with use of these values and compared to the true values.

Table 1. 6

Average elements values

Circuit variant	Element		
	R	$\underline{Z}_L$	X_C
Serial circuit			
Parallel circuit			
Mixed serial and parallel circuit			
Average value:			

Phase shift between current and voltage should be measured with use of oscilloscope in accordance to the two methods given below.

Phase shift measurement with dual channel oscilloscope

In this method one must connect both channels (X and Y) with current and voltage related signal sources. By adjusting the display parameters, one should cover both time axes to obtain similar view as shown in the figure 1.3.

The phase shift angle φ is calculated from the following formula:

$$\varphi = 360° \frac{\Delta x}{x},$$

where: Δx – interval between curves in the time axis intersection, x – interval equivalent to the curves period.

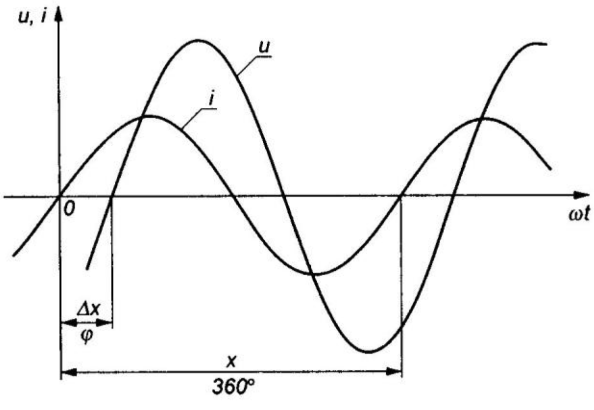

Fig. 1. 3. Illustration of dual channel method

Oscilloscope connection diagram is shown in the figure 1.4.

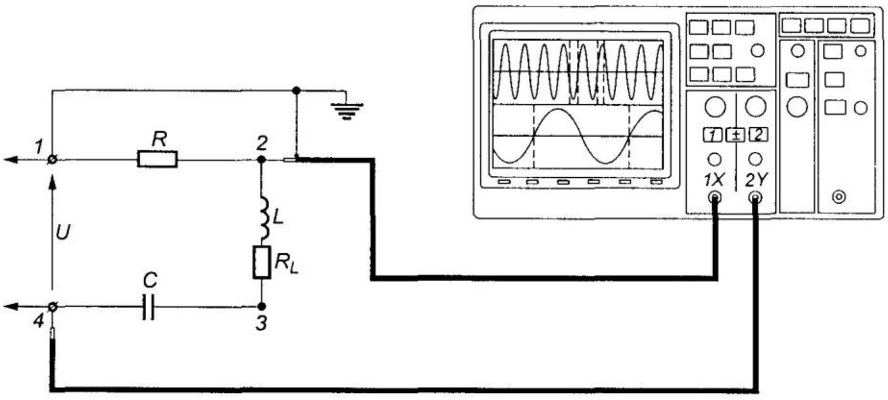

Fig. 1.4 Oscilloscope connection for phase shift measurement

Phase shift measurement using XY presentation

With this method, one needs to measure the geometrical parameters of the ellipse visible after coupling two channels (X and Y) on a single plot. The ellipse will look similar to one shown in fig. 1.5. After the connection of X and Y channels to sinusoidal signals are made, the phase shift angle may be calculated from one of the following formulas:

$$\varphi = \arcsin\frac{a}{b} \qquad \text{or} \qquad \varphi = \arcsin\frac{\alpha\beta}{X_1 Y_1}$$

Figure 1.5 shows how to determine values needed in formulas given above.

In order to make this measurement, the X channel should be giving the voltage plot on the resistance (that is because voltage on the resistance has the same phase as current flowing through it) and the Y channel the voltage on examined element (capacitor or inductor). Both plots should have the same supply common. At last, one needs to switch oscilloscope into XY mode by pressing MAIN-DELAYED button and then select XY option using function button under the screen.

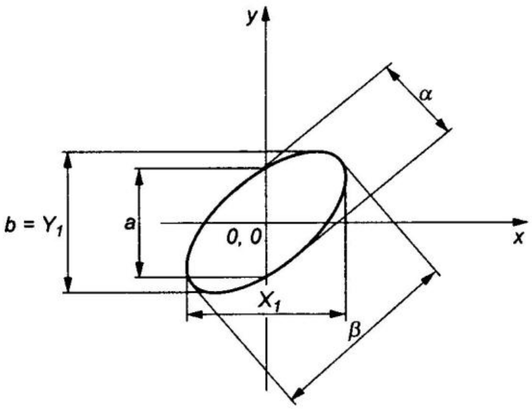

Fig. 1. 5. Illustration of XY method

1.2. CIRCUITS WITH MAGNETIC COUPLING

For this part of exercise one will use magnetically coupled inductors from 3-phase panel. The schematic diagram of the element is shown in the figure 1.6.

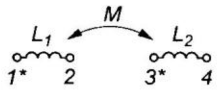

Fig. 1. 6. Shematic diagram of magnetically coupled inductors

The measurements are taken with use of the circuit assembled as shown in the figure 1.7, where U_Z states for phase voltage of 3-phase power supply panel.

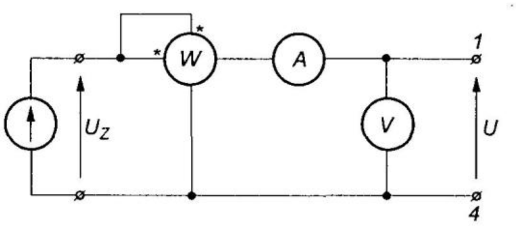

Fig. 1.7. Power supply and meters configuration for measurements in magnetically coupled circuits

The measurements should be taken first for each inductor separately and then for both coils connected in series and coupled in a positive and negative way. The configuration of each circuit with recommended voltage value is shown in figure 1.8.

For the negative coupling (fig. 1.8 d) the additional resistor R=15Ω should be connected in series with voltage source in order to limit a current value while the equivalent inductors impedance is low. Measurements and calculations results of particular configurations given in fig. 1.8 are to be written into a corresponding row of table 1.7.

Table 1. 7

Table of results for magnetically coupled circuits

	Measurements						Calculations												
–	U_Z	P	I	U	U_1	U_2	Z_1	R_1	X_{L1}	L_1	Z_2	R_2	X_{L2}	L_2	Z_W	R_W	L_W	M	k
	V	W	A	V	V	V	Ω	Ω	Ω	H	Ω	Ω	Ω	H	Ω	Ω	H	H	–
a	11										X	X	X	X	X	X	X		X
b	11						X	X	X	X					X	X	X		X
c	44																		
d	22																		

Two coupled inductors examined at point a and b (table 1.7) may be treated as idle state air-core transformer. One may then write:

for point a: $U_2 = \omega M I \Rightarrow M = \dfrac{U_2}{\omega I}$,

for point b: $U_1 = \omega M I \Rightarrow M = \dfrac{U_1}{\omega I}$.

Computed values of mutual inductance are to be written in table 1.7. For point c and d the formulas are:

$$\underline{Z}_W = R_1 + R_2 + j\left(X_{L1} + X_{L2} \pm 2X_M\right) = R_W + jX_W$$

where:

$$Z_W = \frac{U}{I} \quad \text{and} \quad R_W = R_1 + R_2 = \frac{P}{I^2}$$

thus:

$$X_W = \sqrt{Z_W^2 - R_W^2} = X_{L1} + X_{L2} \pm 2X_M$$

Hence one can calculate:

$$X_M = \frac{1}{2}\left[X_W - \left(X_{L1} + X_{L2}\right)\right]$$

and mutual inductance:

$$M = \frac{X_M}{\omega} \quad \text{and} \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

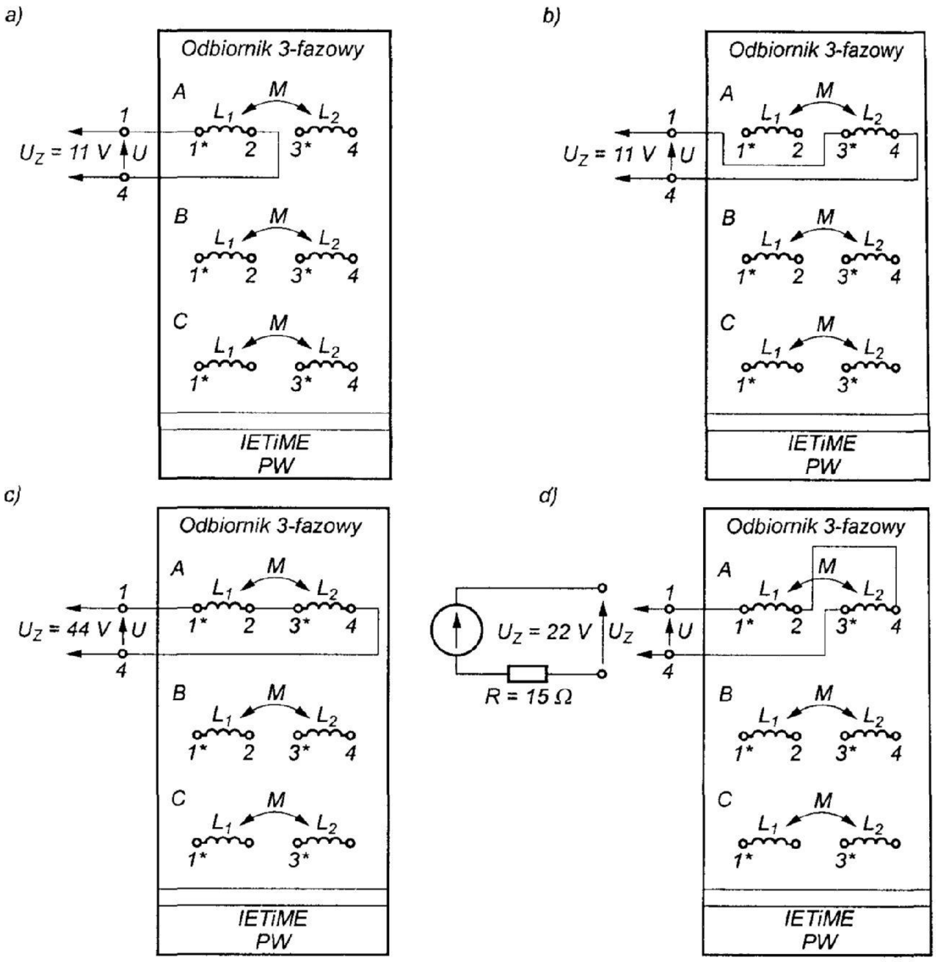

Fig. 1. 8. Inductors configuration diagrams for examining magnetically coupled circuits

2. RESULTS PROCESSING

After running a series of measurements and calculations on *RLC*, *GLC* and mixed serial and parallel circuits one should fill out table 1.8 with obtained results for particular elements in an appropriate configuration.

Table 1. 8

Elements parameters comparation table

Configuration	Serial		Parallel		Mixed serial and parallel	
Results	measured	calculated	measured	calculated	measured	calculated
U_R=						
U_L=						
U_C=						
I=						

Knowing the circuits elements parameters and supply voltage, one should:

1. Calculate theoretical values of voltages and currents for particular elements in all circuit configurations and then make a comparison of measured and calculated current and voltage module values by filling out table 1.8.
2. Work out a conclusions coming from the results given in tables 1.6 and 1.8.
3. Make vector diagrams for each examined circuit.
4. Make vector diagrams for four variants of circuits with magnetic coupling (table 1.7).
5. Draw one's own conclusions.