Laboratory "Modeling and Simulation of Dynamic Systems"
Institute of the Theory of Electrical Engineering, Measurement and Information Systems, WUT

Exercise No 6A

MODELING OF DYNAMIC CHANGES OF SUGAR AND INSULIN IN THE BLOOD

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1. The aim of experiments

Learning the phenomena of changing the temporary level of sugar and insulin in the blood of the healthy and diabetic persons as a result of consumption of the food.

2. Mathematical model

The change of the sugar and insulin in the blood are governed by the following differential equations:

$$\frac{dx_1}{dt} = -a_1 x_1 x_2 + a_2(x_1 - M_1)[1 - sgn(x_1 - M_1)] + b_1 u_1 \quad (1)$$

$$\frac{dx_2}{dt} = a_3(x_1 - M_1)sgn(x_1 - M_1) - a_4 x_2 + b_2 u_2 \quad (2)$$

where the function $sgn(x_1(t) - M_1$ is defined as:

$$sgn(x_1 - M_1) = \begin{cases} 0 & dla \quad x_1 \leq M_1 \\ 1 & dla \quad x_1 > M_1 \end{cases} \quad (3)$$

The Simulink model of these equations is presented in Fig. 1. The typical values of the parameters are as following:

- for healthy person
  $a_1 = 0.05\frac{cm^3}{hmg}, \quad a_2 = 1.0\frac{1}{h}, \quad a_3 = 0.5\frac{1}{h}, \quad a_4 = 2.0\frac{1}{h}, \quad b_1 = b_2 = 1.0\frac{1}{h}$
- for diabetic person
  $a_1 = 0.05\frac{cm^3}{hmg}, \quad a_2 = 1.0\frac{1}{h}, \quad a_3 = (0.0001 - 0.01)\frac{1}{h}, \quad a_4 = 2.0\frac{1}{h}, \quad b_1 = b_2 = 1.0\frac{1}{h}$

Assume $M_1 = 100$. As it is seen the most important difference between the healthy and diabetic person is reflected by the value od the parameter a_3.

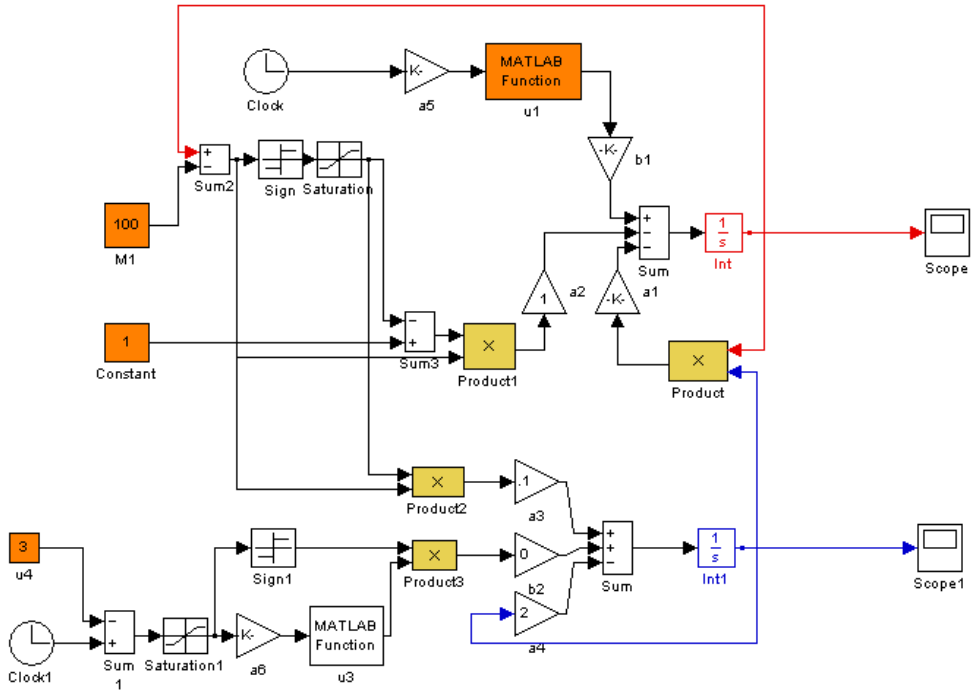

Figure 1: *Simulink model of the dynamic changes of sugar and insulin in the blood*

3. Program of numerical experiments

All experiments are done for the healthy and diabetic persons using the Simulink (file **insulina.mdl**). In particular:

1. Set the parameters corresponding to healthy person. For this model observe the changes of the sugar level $x_1(t)$ and insulin $x_2(t)$ at different initial conditions of $x_1(0)$ and $x_2(0)$ and after consumption of food.

   - initial conditions $x_1(0) = M_1$, $x_2(0) = 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$
   - initial conditions $x_1(0) > M_1$, $x_2(0) > 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$
   - initial conditions $x_1(0) > M_1$, $x_2(0) = 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$

   Set in experiments: $t_0 = 0$, $\tau_1 = 0.6\,h$ and different values of R_1 (for example $0 \leq R_1 \leq 100\frac{mg}{h}$). The value $R_1 = 0$ means lack of food consumption, $R_1 \neq 0$ means some level of food consumption.

2. Set the parameters corresponding to diabetic person. The parameter a_3 might be equal for example ($0.0001 \leq a_3 \leq 0.01$). For this model observe the changes of the sugar level $x_1(t)$ and insulin $x_2(t)$ at different initial conditions of $x_1(0)$ and $x_2(0)$ and after consumption of food.

   - initial conditions $x_1(0) = M_1$, $x_2(0) = 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$
   - initial conditions $x_1(0) > M_1$, $x_2(0) > 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$
   - initial conditions $x_1(0) > M_1$, $x_2(0) = 0$ and $u_1(t) = R_1 e^{-(t-t_0)/\tau_1}$

3. For diabetic person set the initial conditions $x_1(0) > M_1$, $x_2(0) = 0$ and do the following

   - Adjust value of R_2, (representing injection of insulin, $u_2(t) = R_2 e^{-(t-t_0)/\tau_2}$), to provide the proper level of sugar in the blood. Set in experiments $t_0 = 0$ and different values of time constant τ_2 ($0.3 \leq \tau_2 \leq 0.6$).
   - Perform the same experiments at lack of consumption ($R_1 = 0$) and at some determined level of it (the value od R_2 different from zero)
   - Check in experiments different values of delay time t_0 and compare the results to the case of $t_0 = 0$.

4. Discuss and interpret the results of the experiments.