

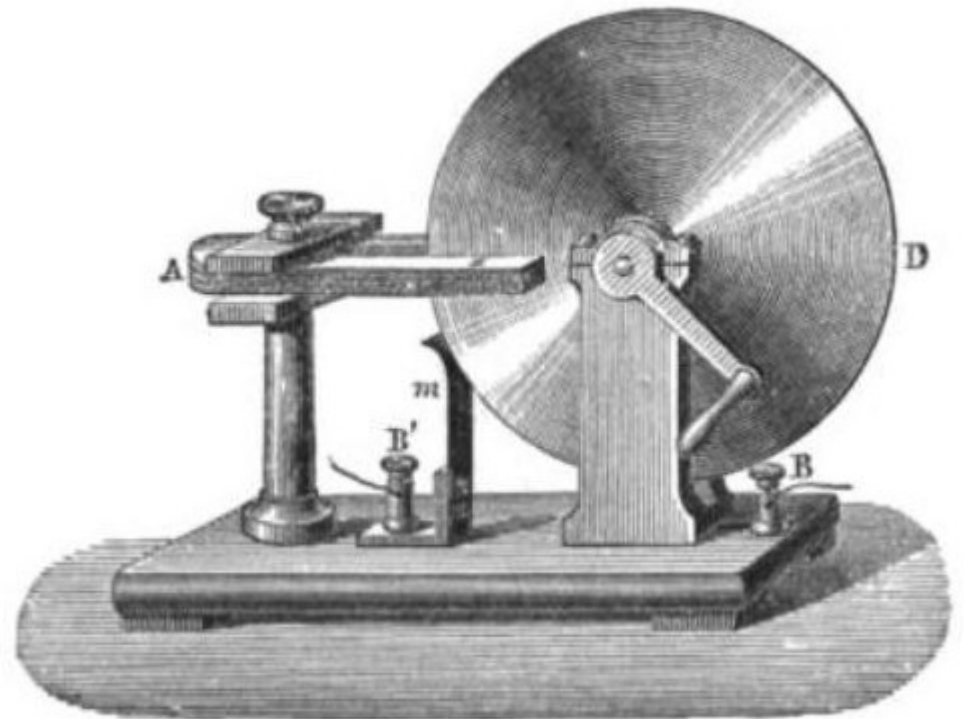
Electromagnetic Fields

Lecture 10

Induction

History

- 1831 - Michael Faraday



Faraday's disk - Rotating a copper disk near a bar magnet with a sliding electrical lead generates DC current.

Magnetic flux

The magnetic flux

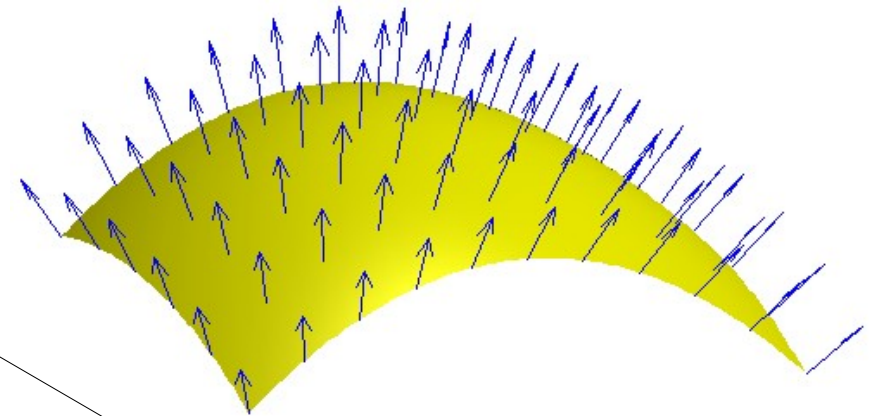
Amount of magnetic field passing through a given surface

$$\iint \mathbf{B} \cdot d\mathbf{S} = \Phi_m$$

Magnetic flux density
(vector field)

Dot product

Surface normal vector



Magnetic flux is measured in Webers [Wb].

Induction

- Faraday's law of induction

The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

$$|\varepsilon| = \left| \frac{d \Phi_m}{d t} \right|$$

Lenz's law

Lenz's law:

- "An induced current is always in such a direction as to oppose the motion or change causing it."

$$\varepsilon = - \frac{d \Phi_m}{d t}$$

Two EMFs

- **Motional EMF** - generated by magnetic force on a moving wire

$$\varepsilon = B v l$$

- **Transformer EMF** - generated by electric force due to a changing magnetic field

$$\varepsilon = B S \omega \cos(\omega t)$$

EMF

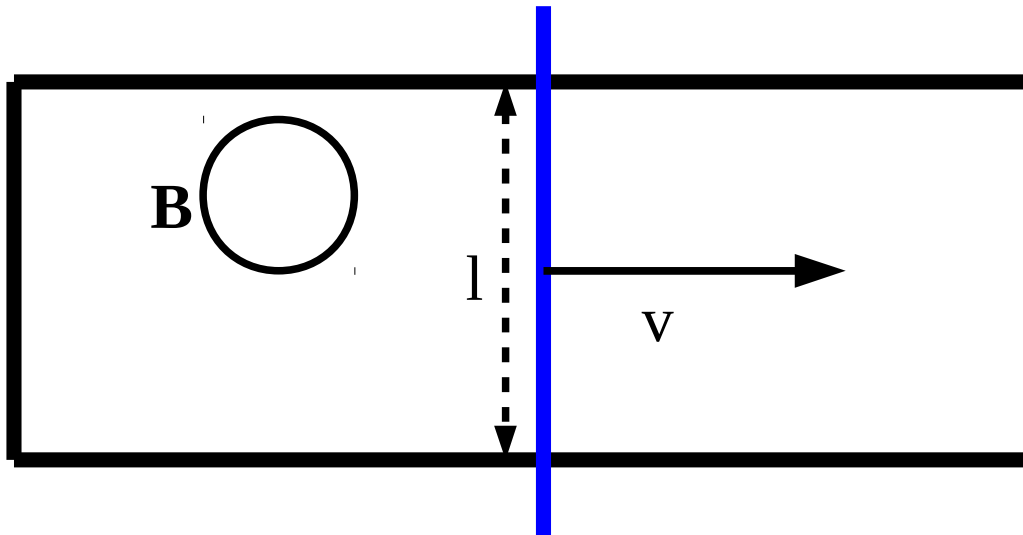
$$\varepsilon = -\frac{d\Phi}{dt} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\varepsilon = -\iint \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

transformer EMF

motional EMF

Case 1: Sliding bar



$$\varepsilon = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = B v l$$

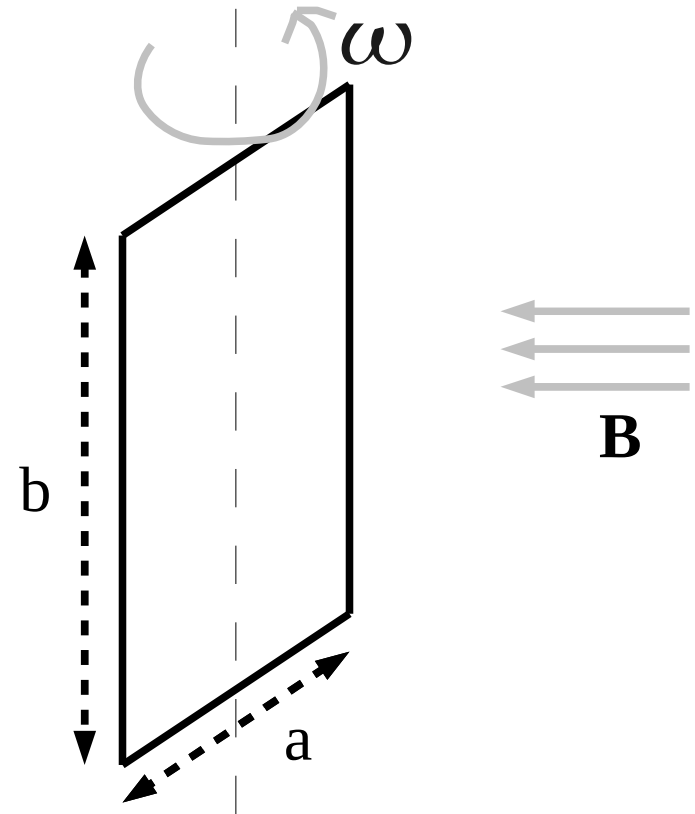
Case 2: Rotating frame

$$\varepsilon = -\frac{d\Phi_m}{dt}$$

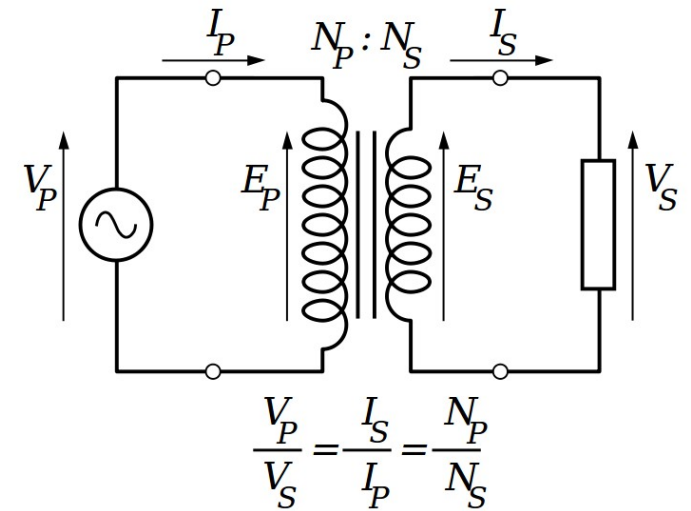
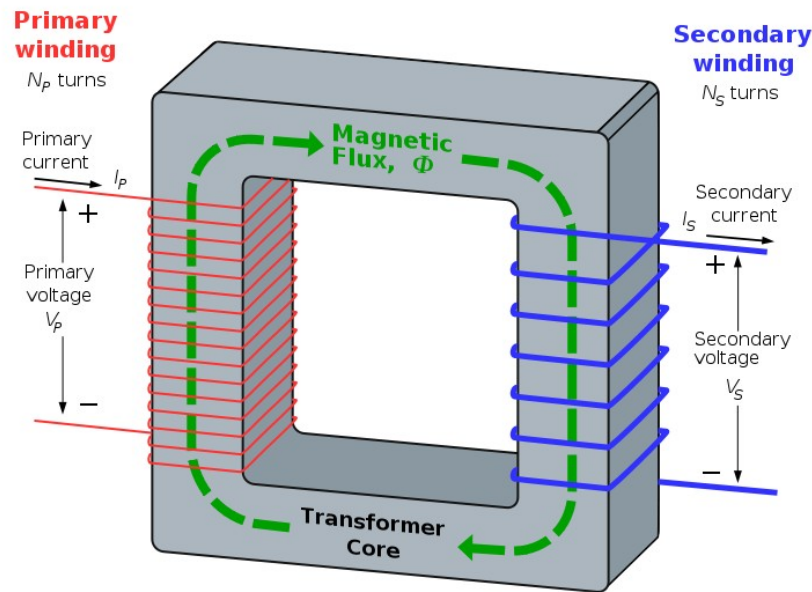
$$\varepsilon = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S}$$

$$\varepsilon = -\frac{d}{dt} B a b \sin(\omega t)$$

$$\varepsilon = -B a b \omega \cos(\omega t)$$



Case 3: Transformer



$$V_p = -N_p \frac{d\Phi}{dt}$$

$$V_s = -N_s \frac{d\Phi}{dt}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

References

References:

Deventra K. Mistry: Practical Electromagnetics, From Biomedical Science to Wireless Communication, Wiley-Interscience, 2007

Joseph F. Becker: Physics 51 - Electricity & Magnetism, California State University
<http://www.physics.sjsu.edu/becker/physics51/>

some figures were taken from Wikipedia.

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