Electromagnetic Fields

Lecture 2

Fundamental Laws

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Laws of what?

Electric field...

is a phenomena that surrounds electrically charged objects or that which is in the presence of a time-varying magnetic field. It exerts a force on other electrically charged objects. The concept of an electric field was introduced by Michael Faraday.

Magnetic field...

is a phenomena that surrounds moving electrically charged objects or that which is in the presence of a time-varying electric field. It exerts a force on other moving electrically charged objects. The concept of an magnetic field was introduced by Michael Faraday.

Lorentz force law:

$$\boldsymbol{F} = q\,\boldsymbol{E} + q(\boldsymbol{v} \times \boldsymbol{B})$$

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Charged particle?

Electric charge...

is a physical property of matter. It is quantized – the elementary charge is called *e* (quarks are believed to have charge being multiple of *e/3*).

$$e \approx 1.602 e^{-19} [C]$$

Charges can be positive and negative. Electron has a charge -*e*, proton +*e*. Charges appeal (different signs) or repell (same signs) each other:

$$\boldsymbol{F} = k_e \frac{q_1 q_2}{r^2}, \qquad k_e = \frac{1}{4 \pi \epsilon_0}$$

Charge conservation law:

The total electric charge of an isolated system remains constant.

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Lorentz force law

$$\boldsymbol{F} = q\boldsymbol{E} + q(\boldsymbol{v} \times \boldsymbol{B})$$

- **LFL** describes the total force acting on a charge in presence of both electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ fields.
- The electric field is a physical property of a matter.
- The magnetic field $\boldsymbol{B}$ is a purely mathematical concept allowing simple calculation of relativistic effects of electricity. It will be shown later that the “magnetic force” can be calculated with special theory of relativity (STR) with no use of magnetic field.
- We shall use $\boldsymbol{B}$, however, because it is simplier.

Maxwell Equations (ME)

- Ampere's law (corrected by Maxwell)
- Faraday's law of induction
- Gauss's law
- Gauss's law for magnetism

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$\boldsymbol{E}$ – electric field (electric field intensity)
$\boldsymbol{D}$ – electric displacement field (electric induction)
$\boldsymbol{H}$ – magnetizing field (magnetic field intensity)
$\boldsymbol{B}$ – magnetic field (magnetic induction)
$\boldsymbol{J}$ – free current density

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$
$$\boldsymbol{J} = \sigma \boldsymbol{E}$$
$$\boldsymbol{B} = \mu \boldsymbol{H}$$

Ampere's circuital law

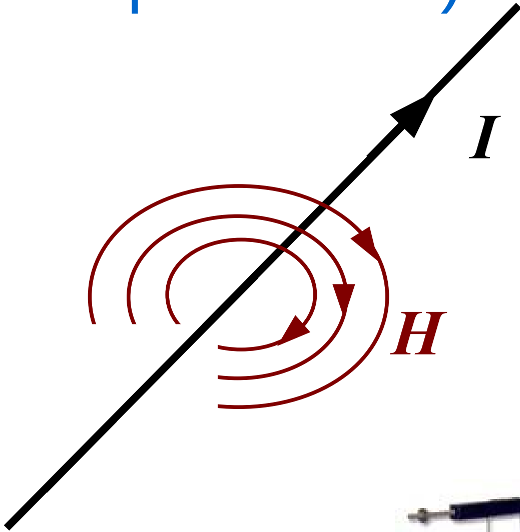
- Ampere's law (Andre-Marie Ampere 1826)
  - differential form

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

  - integral form

$$\oint_{\partial S} \boldsymbol{H}\, d\boldsymbol{l} = \iint_S \boldsymbol{J}\, d\boldsymbol{S}$$

$$\oint_{\partial S} \boldsymbol{H}\, d\boldsymbol{l} = I$$



http://www.sparkmuseum.com

Integral of the magnetic field around a closed loop
is equal to the electric current passing through the loop.

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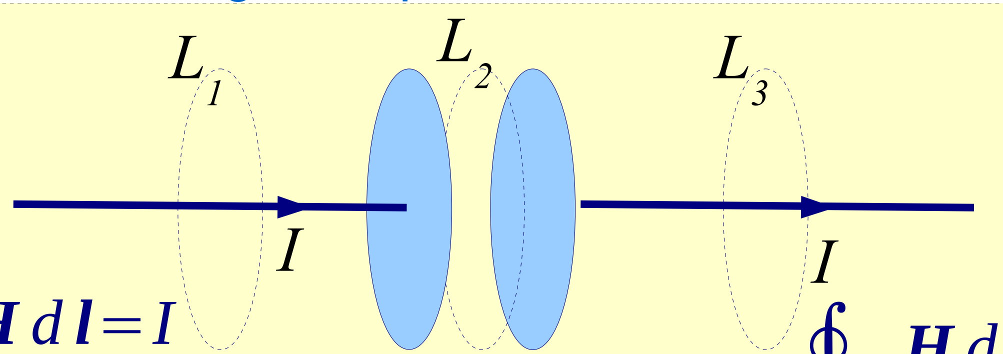
Maxwell-Ampere equation

Ampere's law implies that in a free space, where $\boldsymbol{J}=0$

$$\nabla \times \boldsymbol{H} = 0 \qquad \oint_{\partial S} \boldsymbol{H}\, d\boldsymbol{l} = 0$$

Let us consider the following example



$$\oint_{L_1} \boldsymbol{H}\, d\boldsymbol{l} = I \qquad \oint_{L_3} \boldsymbol{H}\, d\boldsymbol{l} = I$$

$$\oint_{L_2} \boldsymbol{H}\, d\boldsymbol{l} = 0\ ?!?$$

J.C Maxwell suggested correction of the AL

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \oint_{\partial S} \boldsymbol{H}\, d\boldsymbol{l} = \iint_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \right) d\boldsymbol{S}$$

(On Physical Lines of Force, 1861)

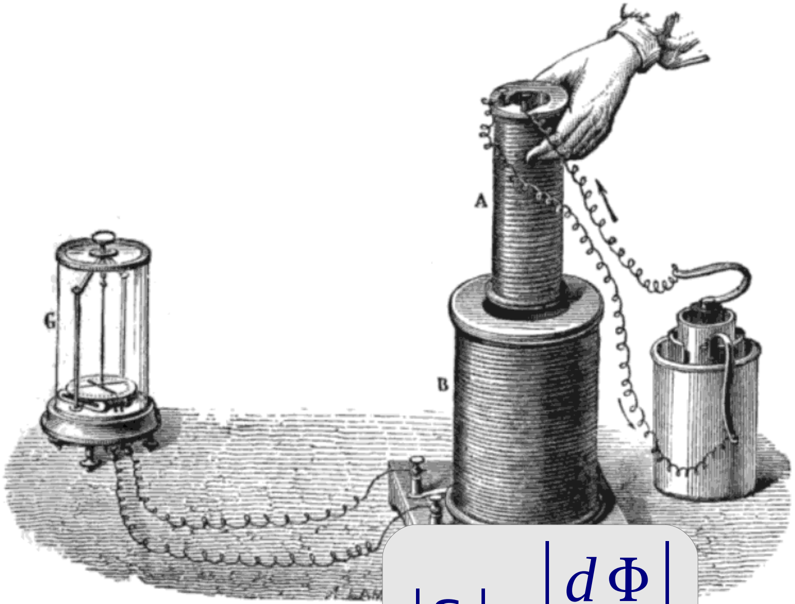
Faraday's law of induction

- Discovered independently by Michael Faraday and Joseph Henry in 1831 (M. Faraday was first to publish)
  - differential form

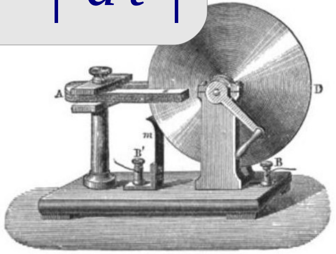
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

  - integral form

$$\oint_{\partial S} \mathbf{E}\, d\mathbf{l} = -\frac{d}{dt}\left(\iint_S \mathbf{B}\, d\mathbf{S}\right)$$



$$|\varepsilon| = \left|\frac{d\Phi}{dt}\right|$$



Electromotive force in a closed loop is equal to the time rate of change of the magnetic flux through the loop.

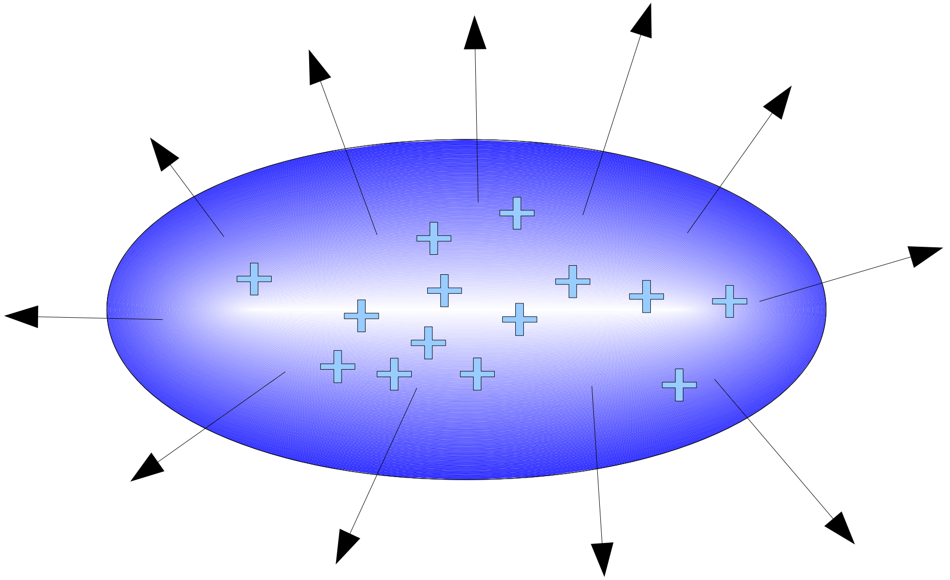
Gauss's law

- Discovered by Carl Friedrich Gauss in 1835 (first published in 1867)
  - differential form

$$\nabla \cdot \boldsymbol{D} = \rho$$

  - integral form

$$\oiint_S \boldsymbol{D}\, d\boldsymbol{S} = Q$$



Electric charge is the source of the electric field.

The total electric flux through the closed surface is equal to the total charge enclosed by the surface.

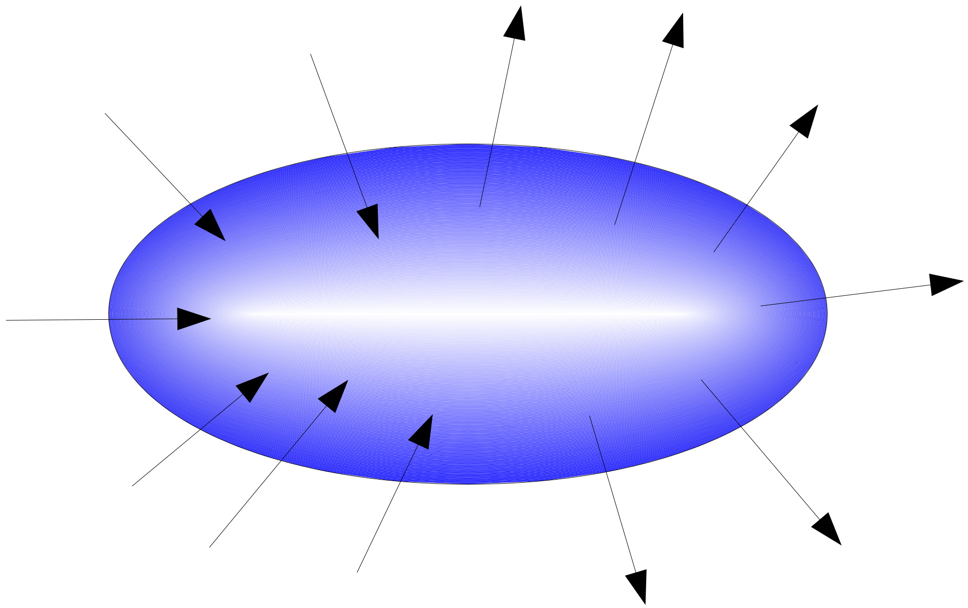
Gauss's law for magnetic field

- Analogy of the Gauss's law for magnetic field
  - differential form

$$\nabla \cdot \boldsymbol{B} = 0$$

  - integral form

$$\oiint_S \boldsymbol{B}\, d\boldsymbol{S} = 0$$



The are no magnetic charges.
(This one is obvious if we remember that $\boldsymbol{B}$ is a pure mathematical concept.)

The total magnetic flux through the closed surface is equal zero.

ME & matter – constitutive relations

In the absence of materials

$$\boldsymbol{D}=\varepsilon_0 \boldsymbol{E}, \quad \boldsymbol{B}=\mu_0 \boldsymbol{H}, \quad \boldsymbol{J}=0 \quad \boldsymbol{E}=0$$

Uniform, linear, isotropic, nondispersive materials

$$\boldsymbol{D}=\varepsilon \boldsymbol{E}, \quad \boldsymbol{B}=\mu \boldsymbol{H}, \quad \boldsymbol{J}=\sigma \boldsymbol{E}$$

The real world

$$\boldsymbol{D}=\varepsilon(\boldsymbol{E},f)\boldsymbol{E}, \quad \boldsymbol{B}=\mu(\boldsymbol{H},f)\boldsymbol{H}, \quad \boldsymbol{J}=\sigma(T(\boldsymbol{J}))\boldsymbol{E}$$

What's more, ε,μ and σ are in general tensors with coefficients dependent upon field strength, direction, frequency and another factors including temperature or mechanical stress, etc.

However, here we will mostly deal with idealized world ☺

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History (1)

J.C. Maxwell, *A Dynamical Theory of the Electromagnetic Field,* 1864

(1) The law ODF total currents $\boldsymbol{J}_{tot}=\boldsymbol{J}+\frac{\partial \boldsymbol{D}}{\partial t}$

(2) The equation of magnetic force $\mu \boldsymbol{H}=\nabla\times\boldsymbol{A}$

(3) Ampere's circuital law $\nabla\times\boldsymbol{H}=\boldsymbol{J}_{tot}$

(4) Electromotive force $\boldsymbol{E}=\mu\,\boldsymbol{v}\times\boldsymbol{H}-\frac{\partial \boldsymbol{A}}{\partial t}-\nabla\varphi$

(5) The electric elasticity equation $\boldsymbol{E}=\frac{1}{\varepsilon}\boldsymbol{D}$

(6) Ohm's law $\boldsymbol{E}=\frac{1}{\sigma}\boldsymbol{J}$

(7) Gauss's law $\nabla\cdot\boldsymbol{D}=\rho$

(8) Equation of continuity $\nabla\cdot\boldsymbol{J}=-\frac{\partial\rho}{\partial t}$ or $\nabla\cdot\boldsymbol{J}_{tot}=0$

J.C.Maxwell has written 6 equations in scalar notation for cartesian system of coordinates, getting 20 equations with 20 unknowns. Here the equivalent equations are written in modern, vector notation.

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History (2)

J.C. Maxwell, *A Treatise on Electricity and Magnetism,* 1873

$$\boldsymbol{E} = -\nabla \varphi - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J}$$

Potentials

$$\nabla \cdot \boldsymbol{B} = 0$$

↓

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

↓

$$\nabla \times \boldsymbol{E} = -\nabla \times \left(\frac{\partial A}{\partial t} \right)$$

↓

$$\nabla \times \left(\boldsymbol{E} + \frac{\partial A}{\partial t} \right) = 0$$

↓

$$\boldsymbol{E} + \frac{\partial A}{\partial t} = -\nabla \varphi$$

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Practical use

- ME allow us to predict general behavior of electromagnetic fields
- Practical problems need finite restriction in time and space
- We need boundary conditions to restrict domain of interest
- We need initial conditions to restrict time of interest

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Conditions? What for?

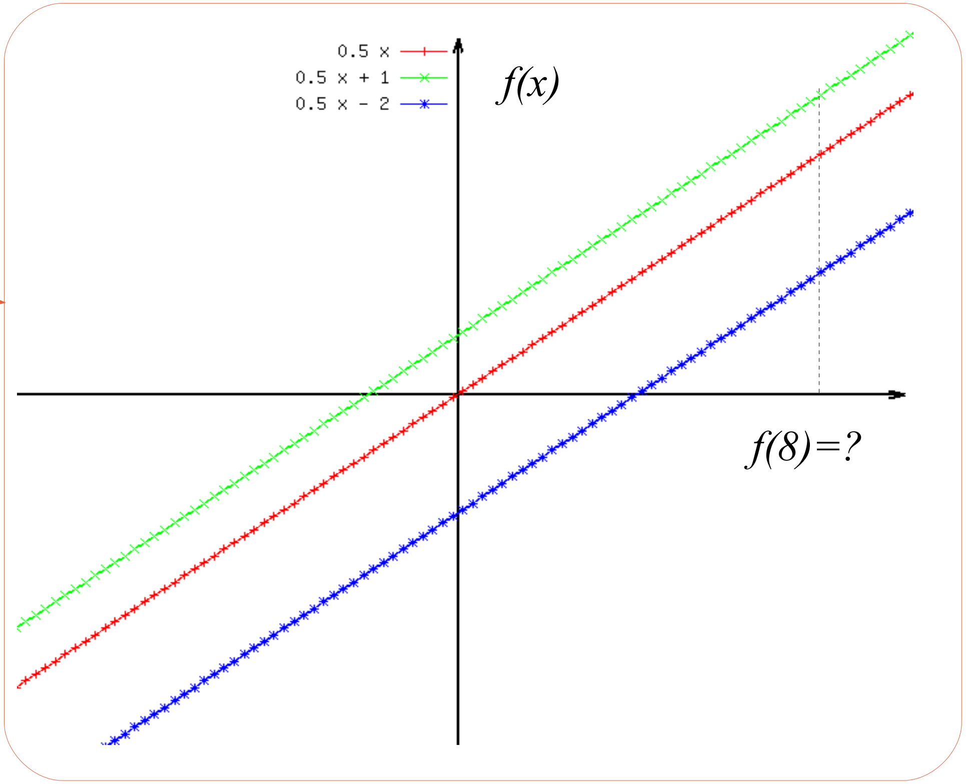
ME are *partial differential equations* – they *specify* field *derivatives*, not the fields itself.

Let's consider a simple example of ordinary DE:

$$\frac{d f(x)}{d x} = 0.5$$

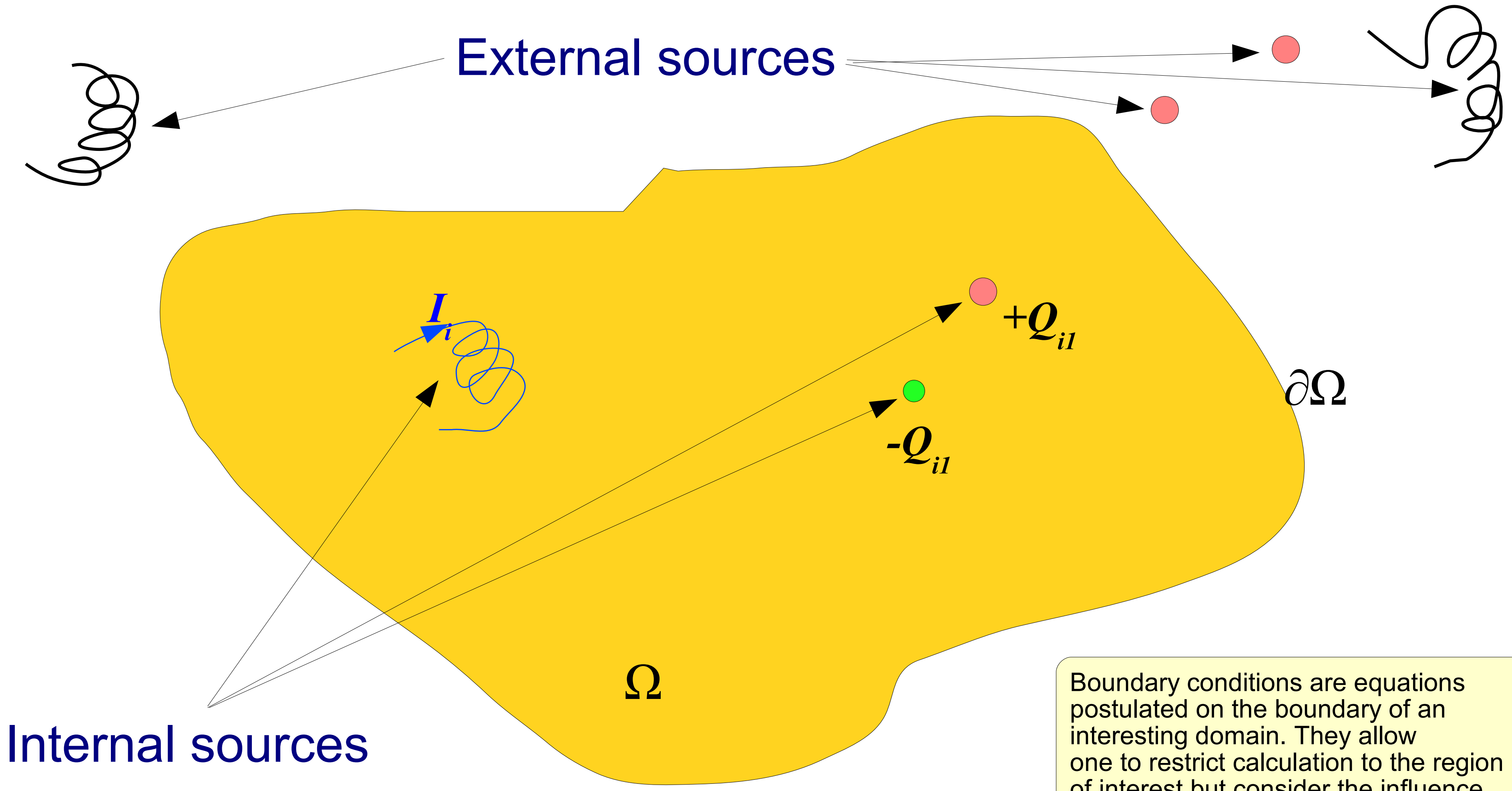
$$f(x) = 0.5 x + c$$

$c = ?$



As you can see, a general solution of DE gives us an information on the character (type) of function, but we need additional equations to choose the particular solution.

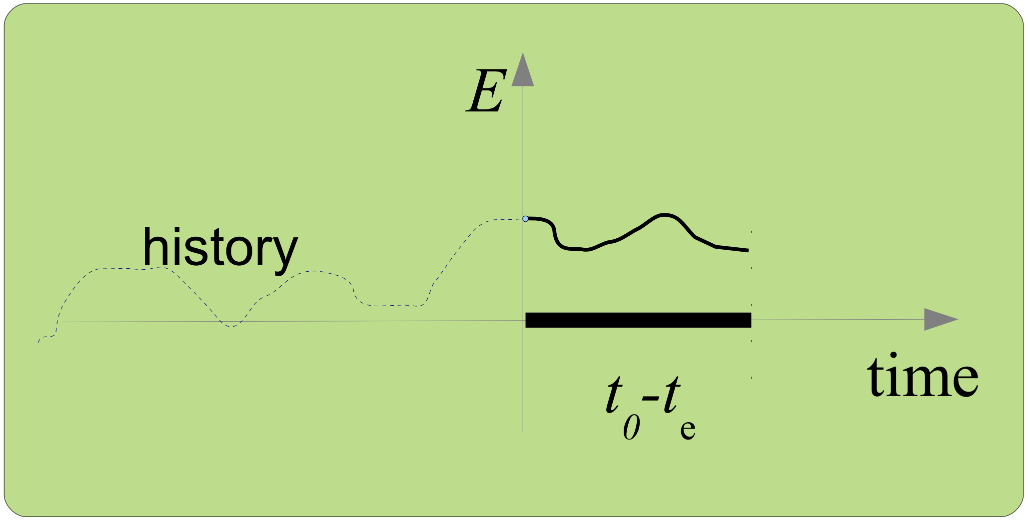
Boundary conditions

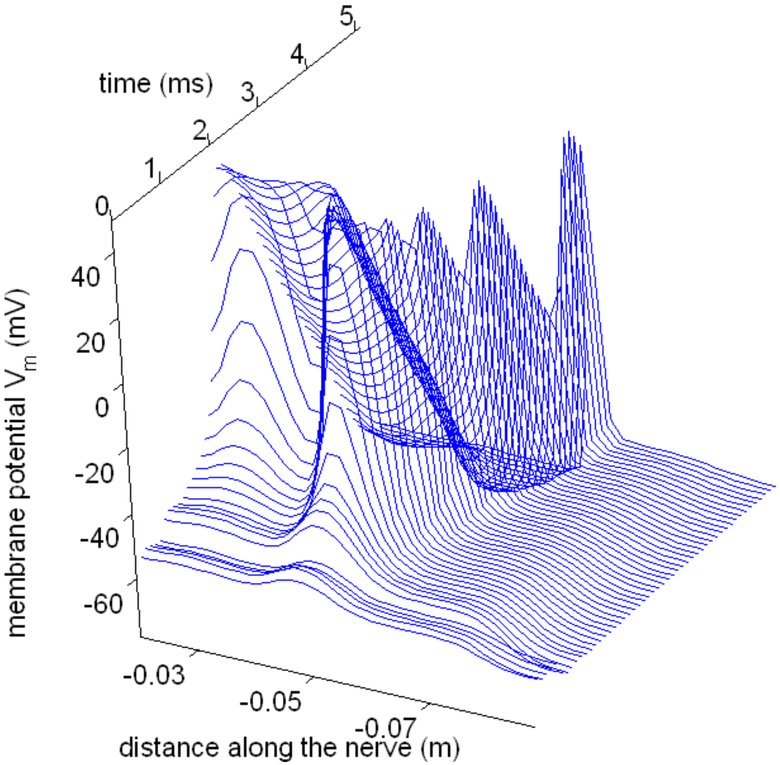


Boundary conditions are equations postulated on the boundary of an interesting domain. They allow one to restrict calculation to the region of interest but consider the influence of external sources.

Initial conditions

Initial conditions are equations postulated at some point in time. They allow one to restrict calculation to the limited period but consider the influence of the system's history.





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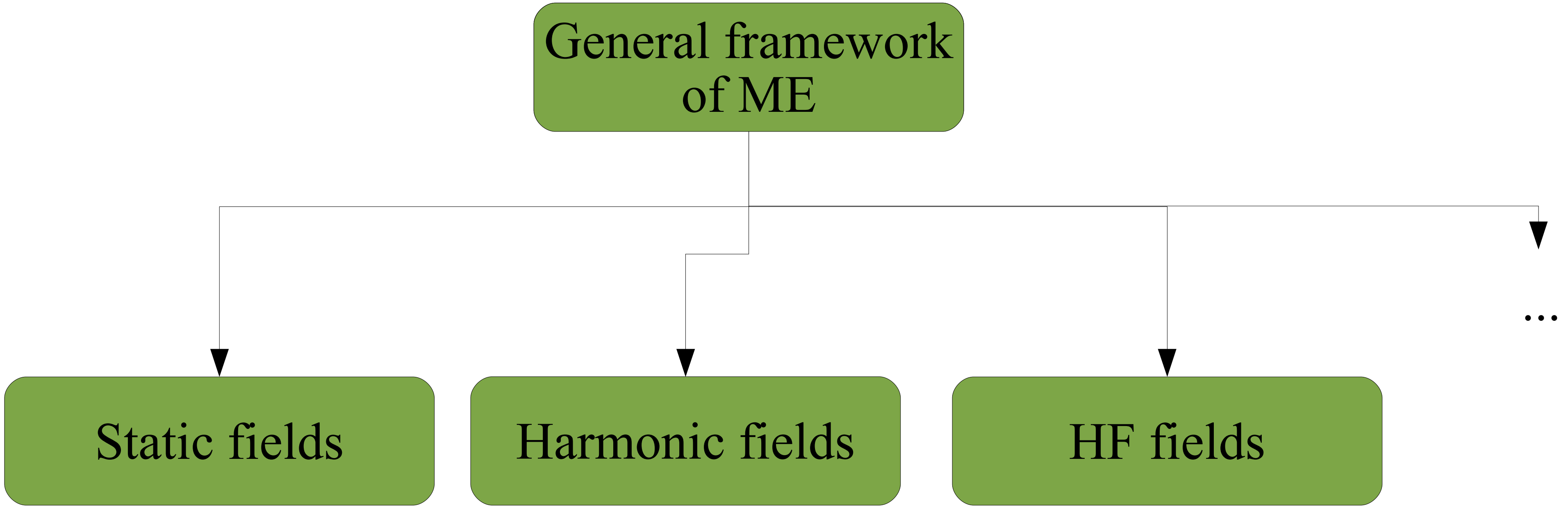
Simplifications

ME are very general and quite complicated. Luckily quite often we can simplify model of the problem being considered.
To do so we neglect some phenomena which are weak, exploit some symmetries or use specific way of mathematical description.



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Simplifications: electrostatics

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

We are interested in phenomena arisen from stationary or *very* slow moving charges.

$$\frac{\partial \boldsymbol{B}}{\partial t} = 0, \qquad \frac{\partial \boldsymbol{D}}{\partial t} = 0$$

Only electric field of interest.
There are no charge sources.

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

$$\nabla \times \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

We may use scalar, not vector !!

$$\boldsymbol{E} = \nabla \varphi$$

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Simplifications: magnetostatics

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

We are interested in magnetic phenomena arisen from magnets and steady or direct currents.

$$\frac{\partial \boldsymbol{B}}{\partial t} = 0, \quad \frac{\partial \boldsymbol{D}}{\partial t} = 0$$

Only magnetic field of interest.

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

$$\nabla \times \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

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