# Electromagnetic Fields

*Lecture 6*

# *Magnetostatics 1*

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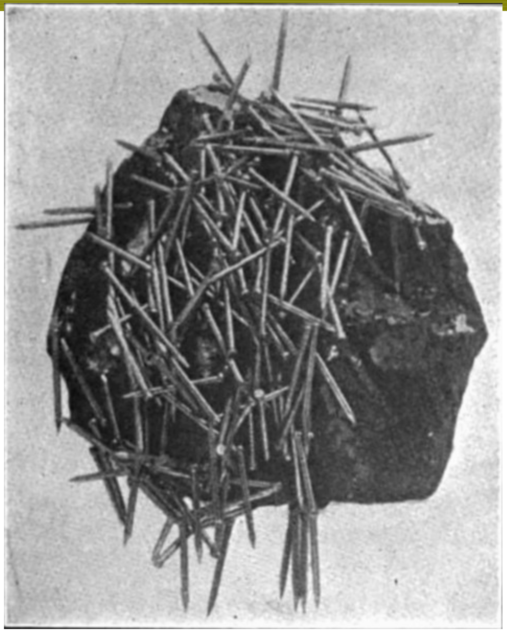
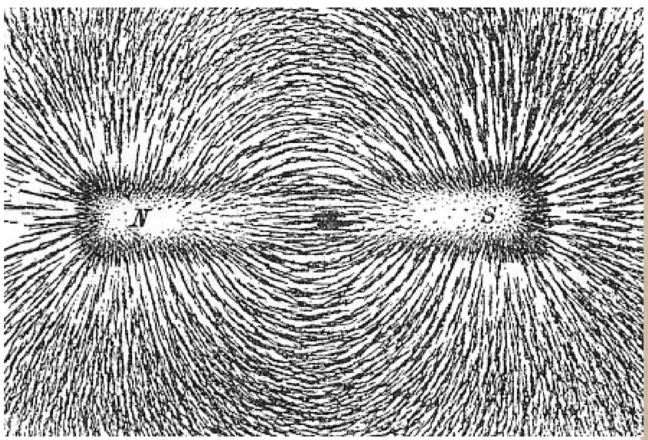
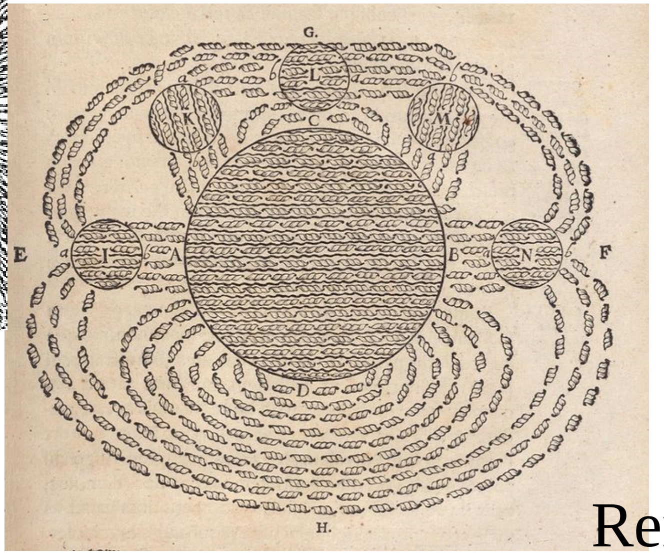
# History

- Lodestone: natural magnets
- Magnetic compasses



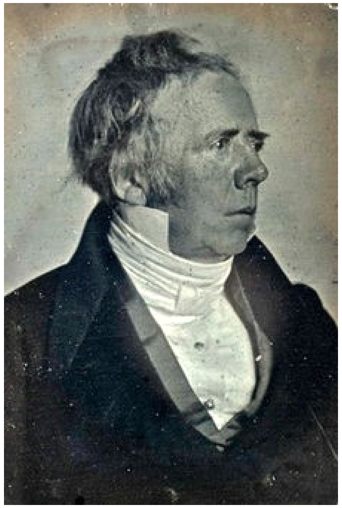


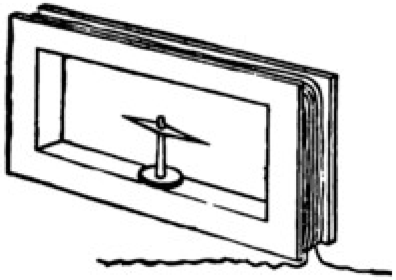

René Descartes, 1644.

# History

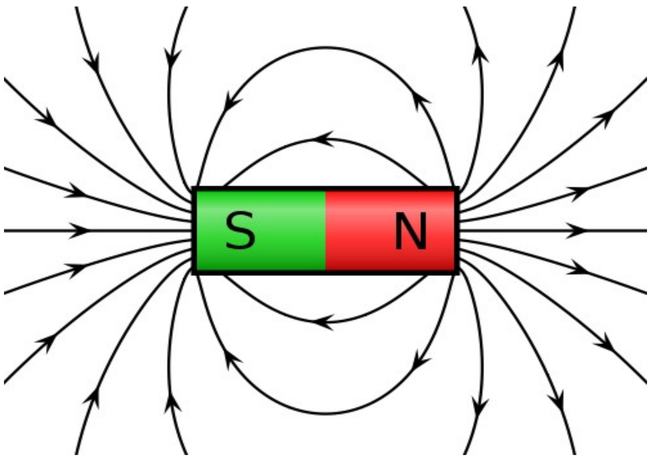
- Hans Christian Ørsted (Oersted) 1777-1851



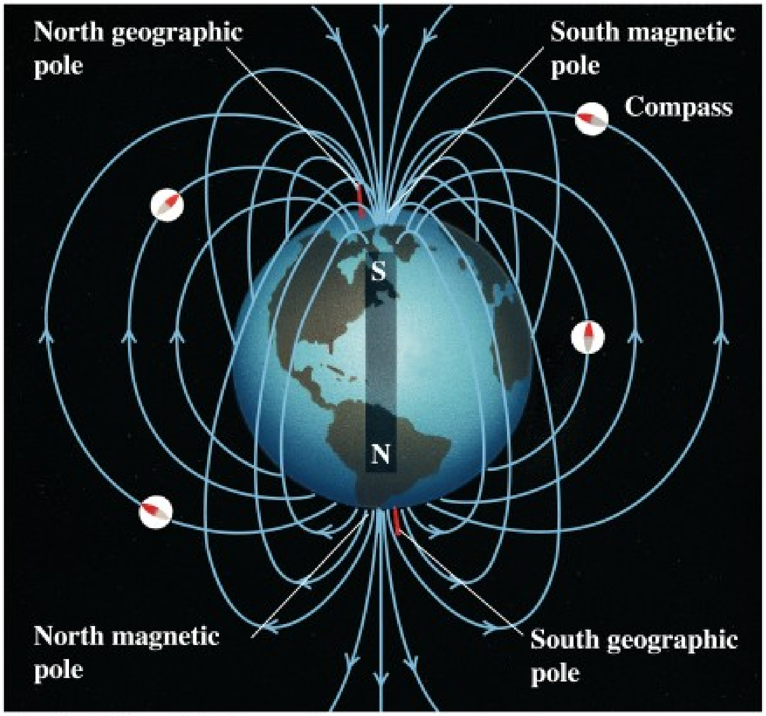
- André-Marie Ampère 1775-1836)



# Magnetic Field of the Earth



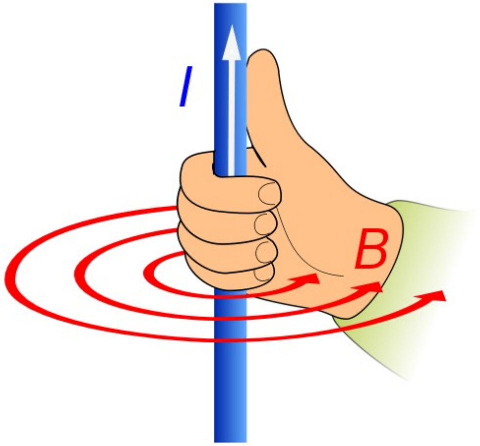

[Wikipedia]

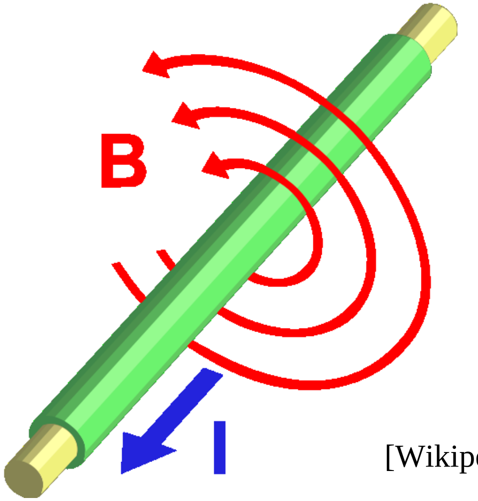




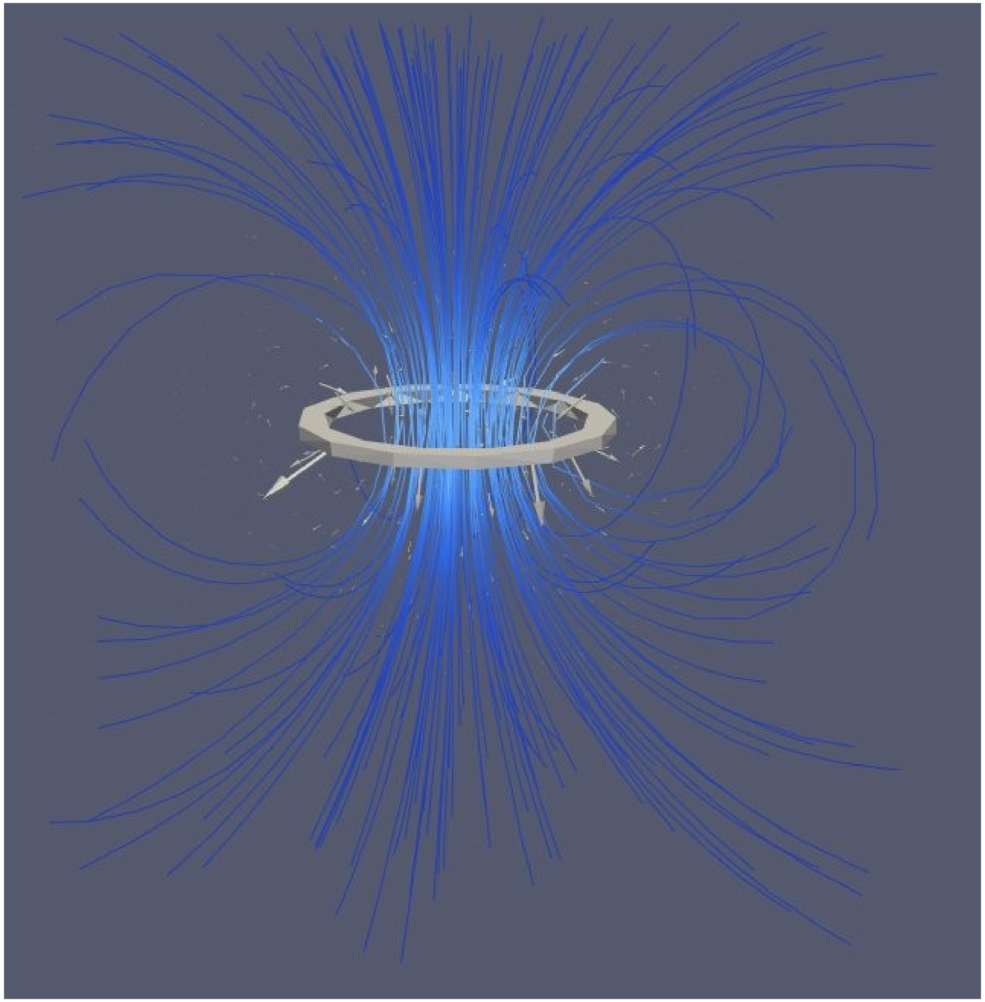

North geographic pole = South magnetic pole

# Right hand grip rule




[Wikipedia]



# Magnetic field

- Magnetic field intensity

$$\boldsymbol{H}$$ [A/m] - Amper per metre

- Magnetic flux density

$$\boldsymbol{B}=\mu\,\boldsymbol{H}$$ [T] - Tesla

$$\boldsymbol{B}=\mu_0(\boldsymbol{H}+\boldsymbol{M})$$

magnetic permeability

$$\boldsymbol{B}=\mu_0(1+\chi_m)\boldsymbol{H}=\mu\,\boldsymbol{H}$$

magnetic susceptibility

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# Ampere's Law

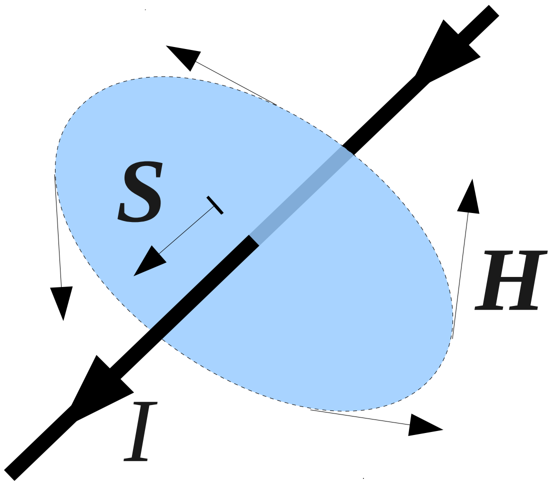
- Ampère's Circuital Law

The integrated magnetic field around a closed loop is equal the electric current passing through the loop.

$$\oint_C \boldsymbol{B} \cdot d\,\boldsymbol{l} = \mu_0 I$$

$$\oint_C \boldsymbol{H} \cdot d\,\boldsymbol{l} = I$$

$$\oint_C \boldsymbol{H} \cdot d\,\boldsymbol{l} = \int_S \boldsymbol{J} \cdot d\,\boldsymbol{S}$$



# Gauss's law for magnetism

Magnetic monopoles does not exist.

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\oint \boldsymbol{B} \cdot d\,S = 0$$

There isn't any point where magnetic isolines starts.
All of them are closed loops.

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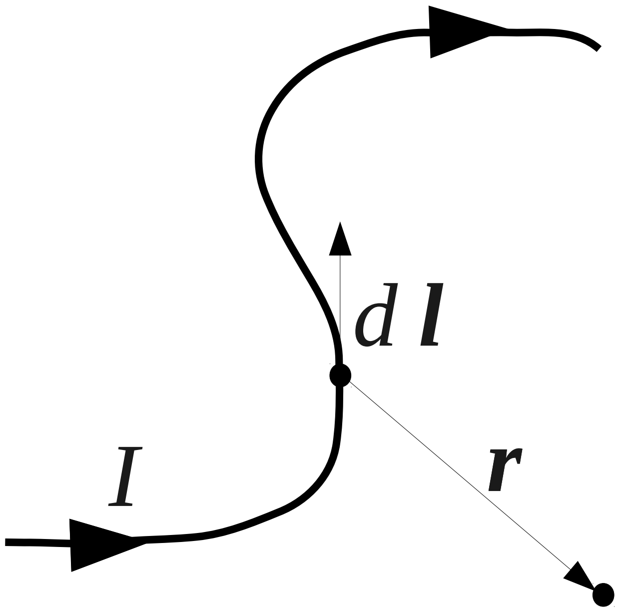
# Biot-Savart Law

The Biot–Savart law is used to compute the magnetic field generated by a steady current flowing in the wire.

$$\boldsymbol{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\boldsymbol{l} \times \boldsymbol{r}}{|r|^3}$$




I - electric current,
**dl** - a vector, whose magnitude is the length of the differential element of the wire, and whose direction is the direction of current,
**B** - magnetic field,
**u0** - magnetic constant,
**r** - displacement vector,
|r| - magnitude of **r**,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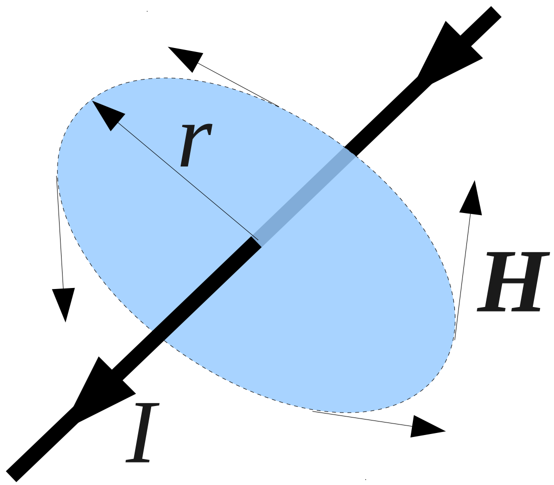
# Example 1: long wire

Find value of magnetic field intensity around long wire with current I.




Ampere's law:

$$\oint_C \boldsymbol{H} \cdot d\,\boldsymbol{l} = I$$

Because of symmetry, H is constant for given r, so:

$$H \oint_{\text{circle}} 1\,dl = I$$

$$H\,2\pi\,r = I$$

$$H = \frac{I}{2\pi\,r}$$

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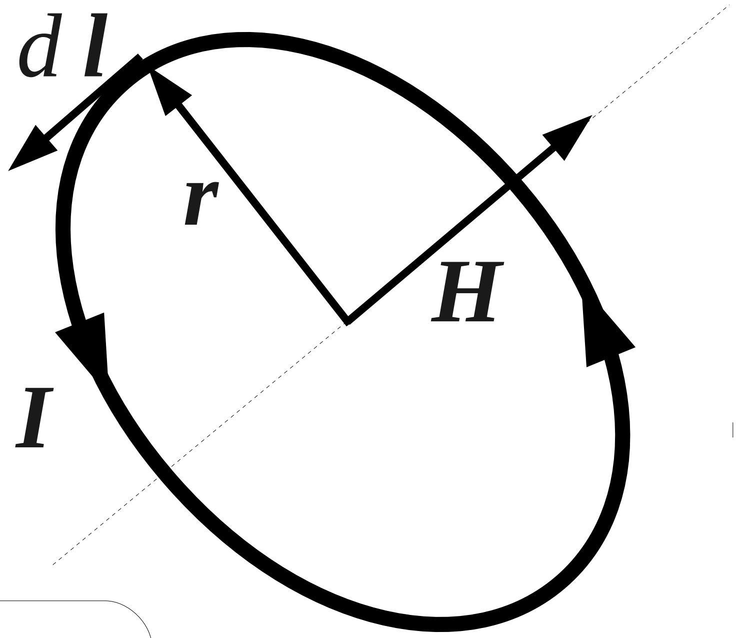
# Example 2: single loop

Find value of magnetic field intensity in the center of circular wire loop with current I.

Biot-Savarte law:

$$\boldsymbol{H} = \int \frac{1}{4\pi} \frac{I\, d\boldsymbol{l} \times \boldsymbol{r}}{|r|^3}$$




For the center of circle, **r** and angles are constant, so:

$$H = \frac{I}{4\pi r^2} \int_{\text{circle}} 1\, d l$$

$$H = \frac{I}{2r}$$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# Energy in magnetics

- Energy density: $$u = \frac{\boldsymbol{B} \cdot \boldsymbol{H}}{2}$$

- Total energy stored in the field: $$U = \int_V u \, dv$$

- For coil: $$U = \frac{1}{2} L I^2$$

# Example 3: energy in wire

Find energy stored in the magnetic field in one meter of long, straight coaxial cable.

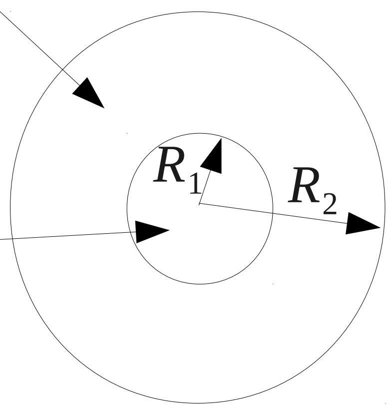
$$u=\frac{\mathbf{B}\cdot\mathbf{H}}{2}$$

$$u_1=\frac{\mu_0 J^2 r^2}{4}$$

$$u_2=\frac{\mu_0 J^2 \pi^2 R_1^4}{8\pi^2 r^2}$$

$$H_2=\frac{J\pi R_1^2}{2\pi r}$$

$$H_1=\frac{J r}{2}$$




$$U=U_1+U_2=\int_{r=0}^{R_1} u_1\,\mathrm{d}v+\int_{r=R_1}^{R_2} u_2\,\mathrm{d}v$$

# Example 3: cont.

$$U_1 = \frac{\mu_0 \pi J^2}{2} \int_{r=0}^{R_1} r^3 \, \mathrm{d}r \qquad U_2 = \frac{\mu_0 J^2 \pi^2 R_1^4}{4\pi} \int_{r=R_1}^{R_2} \frac{1}{r} \, \mathrm{d}r$$

$$U_1 = \frac{\mu_0 \pi J^2}{8} R_1^4 \qquad U_2 = \frac{\mu_0 J^2 \pi R_1^4}{4} \ln\left(\frac{R_2}{R_1}\right)$$

Total energy: $U = U_1 + U_2$

# References

**References:**

Deventra K. Mistra: Practical Electromagnetics, From Biomedical Science to Wireless Communication, Wiley-Interscience, 2007

Joseph F. Becker: Physics 51 - Electricity & Magnetis, Califonia State University
http://www.physics.sjsu.edu/becker/physics51/

*some figures were taken from Wikipedia.*

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