Electromagnetic Fields

Lecture 9

Forces

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Forces in EMF

- Lecture Outline:
  - Electrostatic force
  - Magnetic force
  - Lorentz force
  - Maxwell Tensor
  - Potential Forces

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Electrostatic force

Force is used to define electric field:
(for probe point charge Q)

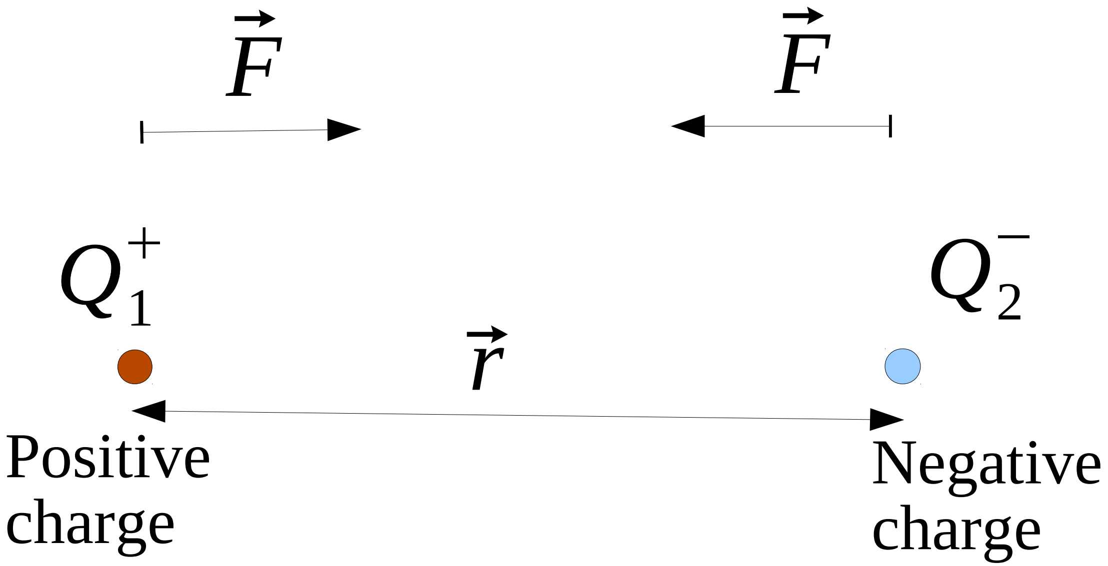
$$\boldsymbol{F} = \boldsymbol{E}\,Q$$

Direction of the force is the same as direction of the electric field.

For charge density, we need to integrate:

$$\boldsymbol{F} = \int \boldsymbol{E}\,\rho\,dv$$

Electrostatic force



$$\vec{F} = \frac{Q_1 Q_2}{\varepsilon 4 \pi r^2} \vec{1}_r$$

- Opposite charges are attracting each other.
- Like charges repel one another.

Magnetic force

Magnetic field is interacting with moving charge (speed v):

$$\boldsymbol{F} = Q(\boldsymbol{v} \times \boldsymbol{B})$$

$$I = \frac{dQ}{dt}$$

$$v = \frac{dl}{dt}$$

We could use electric current (I):

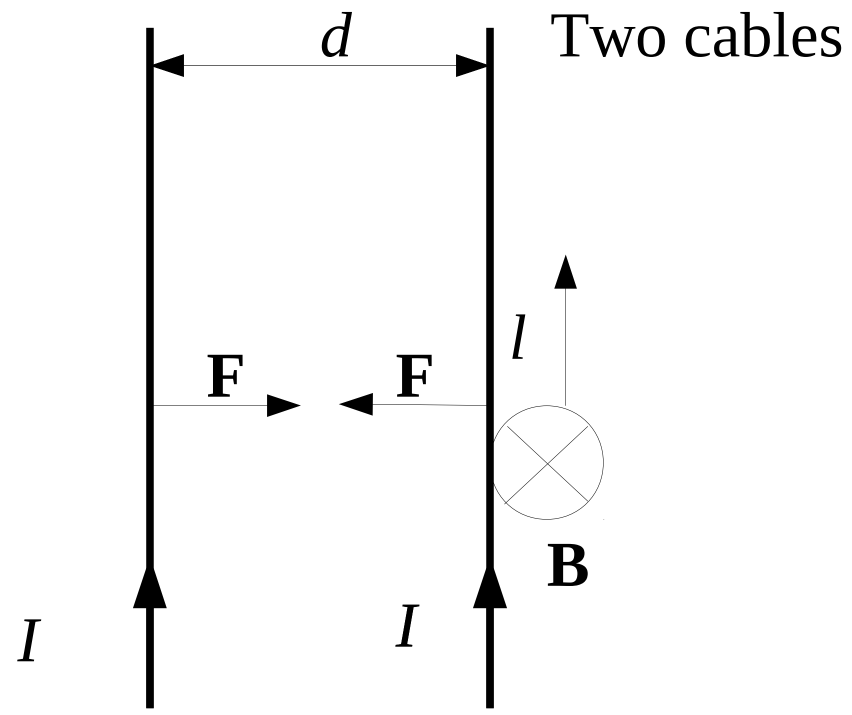
$$\boldsymbol{F} = Q(\frac{dl}{dt} \times \boldsymbol{B}) = \frac{dQ}{dt}(\boldsymbol{l} \times \boldsymbol{B}) = I(\boldsymbol{l} \times \boldsymbol{B})$$

Magnetic force

$$\boldsymbol{F} = I\,(\boldsymbol{l} \times \boldsymbol{B})$$

$$B = \frac{\mu I}{2\pi d}$$

$$F = \frac{\mu I^2 l}{2\pi d}$$



- Opposite currents repel one another.
- Like currents attract each other.

Lorentz Force

The Lorentz force is the force on a point charge due to electromagnetic fields.

$$\boldsymbol{F} = Q\left(E + (\boldsymbol{v} \times \boldsymbol{B})\right)$$

Magnetic flux denisity

Charge

Velocity of conductor

Electric field

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Continuum Forces

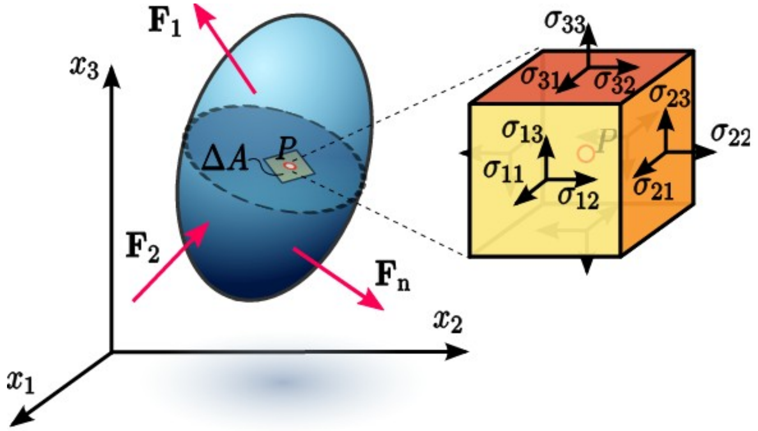
Pressure

$$P = \frac{dF_n}{dS}$$

Pressure is the force per unit area applied in a direction perpendicular to the surface of an object.

Stress tensor

$$\sigma = \frac{dF}{dA}$$



Stress is a measure of the internal forces acting within a object.

Maxwell Stress Tensor

Force density field [N/m3]

$$\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}$$

Poynting vector

$$\mathbf{f} + \epsilon_0\mu_0\frac{\partial\mathbf{S}}{\partial t} = \nabla\cdot\mathbf{T}$$

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\left(\varepsilon_0 E^2 + \tfrac{1}{\mu_0} B^2\right)\delta_{ij}$$

Maxwell Stress Tensor

Kronecker delta

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Potential energy

Potential energy - Energy stored in the system of objects due to its positions.

$$W = -\Delta U = \int \boldsymbol{F} \cdot d\,\boldsymbol{r}$$

$$\boldsymbol{F} = -\frac{d\,U}{dx}$$

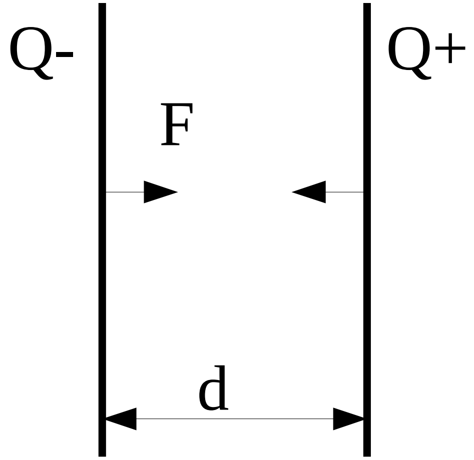
Conservative force

Potential force

Example: Force compressing capacitor

$$W = \frac{Q^2}{2C} \qquad C = \frac{\varepsilon s}{d} \qquad \longrightarrow \qquad W = \frac{Q^2 d}{2 \varepsilon s}$$

$$\boldsymbol{F} = -\frac{dW}{dd} = -\frac{Q^2}{2 \varepsilon s}$$



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